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EXPERIMENTAL AND NUMERICAL MODELING OF THE THERMAL CONDUCTIVITY OF HIGH-VISCOSITY HYDROCARBON SYSTEMS

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A mathematical model is developed to numerically predict the heating of heavy hydrocarbon systems. A comparative analysis of numerical and experimental data is performed. It is found that the thermal conductivity of a hydrocarbon system under study heated from an initial temperature of 24°C *to* 100°C *increases by a factor of* 40 *and, with allowance for free convection, an additional substantial* (*up to* 16 *times*) *increase in heat transfer due to enhanced effective thermal conductivity is observed.*

Key words: *thermal conductivity, free convection, complex heat transfer, induction heating, hydrocarbon fluid.*

Introduction. The thermal conductivity of dropping liquids ranges from 0.08 to 0.7 W/(m·K) and is normally believed to exhibit only slight changes with temperature [1]. It is known, however, that various convective flows and turbulization regimes can arise in a fluid under sufficiently high temperature gradients. In such cases, the effective thermal conductivity of the fluid becomes much higher than its molecular value. In the thermal aspect, as a result, such a fluid is an analog of a solid with a rather high thermal conductivity. The effective thermal conductivity of fluids is estimated as [2] $960 < \lambda_{\rm eff}/\lambda < 1500$, where λ and $\lambda_{\rm eff}$ are the molecular and effective thermal conductivities, respectively.

Turbulent flows are known to display fast irregular changes in velocity, which cause rather intense mixing of the flow [3]. As a result, turbulent mixing becomes responsible for the mass, momentum, and energy fluxes, molecular transfer being negligible on the background of turbulent mixing. Ostroumov and Tetyuev [4] experimentally examined convection in inclined cylindrical glass ducts filled by water and discovered a sharp threshold phenomenon: on reaching a certain intensity of heating from the lower end of the duct and on establishment of a certain temperature gradient, a noticeable flow suddenly originated in the fluid. In this situation, the axial heat transfer increases and can exceed the molecular heat transfer by a factor of 2500.

In the experiments described below, we observed a drastic increase in effective thermal conductivity in heated high-viscosity hydrocarbon fluids, which are almost nonfluid and weakly conducting heat at low temperatures. Examples of such fluids are oil asphalt, high-viscous oil, products of the so-called oil-sludge storehouses, and residual petroleum-refinement products deposited on the bottom of reservoirs.

Experimental Study. Laboratory experiments were performed to model a reservoir filled by a hydrocarbon fluid (oil-sludge), which could be heated by a specially designed induction heater (Fig. 1).

The experimental model consisted of a metal reservoir with a 40-mm thick concrete bed at the bottom. The reservoir, 470 mm high and 450 mm in diameter, was filled by an initially nonfluid high-viscosity hydrocarbon fluid to a height of 350 mm from the concrete-bed surface. The height of the inductor was 250 mm, and its diameter

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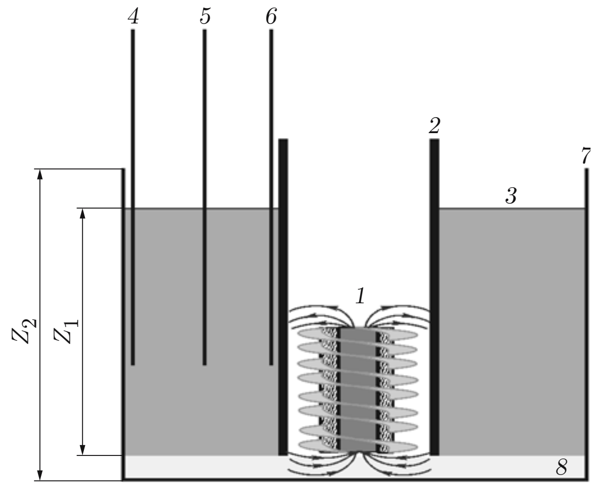

Fig. 1. Schematic diagram of the experimental setup: inductor coil (1), inductor tube (2), hydrocarbon fluid (3), thermocouples (4–6), reservoir (7), and concrete (8).

was 128 mm. The height of the inductor tube was 500 mm, its inner diameter was 148 mm, and its outer diameter was 160 mm. The inductor tube was installed at the centerline of the reservoir, towering 150 mm above the surface of the hydrocarbon fluid. In the course of the experiment, the inductor was fed by standard electric mains with a voltage of 220 V and frequency of 50 Hz. The output power was $N_0 = 1224$ W. The temperature was measured by three thermocouples located at a height of 170 mm from the reservoir bottom and at distances of 10, 60, and 110 mm from the inductor-tube wall. The readings of the thermocouples were recorded every five minutes. The initial temperature was $T_0 = 24°$C.

The experimental data are plotted in Fig. 2, where the registered values of temperature are shown by points.

The molecular thermal conductivity at the initial temperature is low. As a consequence, only the first thermocouple registers a noticeable increase in temperature at the beginning of heating: the temperature rose to 28°C in 5 min and to 38°C in 10 min; then, the temperature increased with some acceleration to 93°C, after which the temperature almost ceased to increase. The temperature at the position of the second thermocouple started to noticeably increase 35 min after the beginning of heating, when the readings of the first thermocouple were already stabilized. The increase in temperature at the position of the second thermocouple is smoother, because the heat from the inductor was transmitted to a larger volume of the medium. An increase in readings of the third thermocouple was observed 140 min after the beginning of heating, and it was smoother than the increase registered by the second thermocouple. The data in Fig. 3 depict a very strong temperature dependence of effective thermal conductivity of oil sludge [1, 5] and suggest that the effective thermal conductivity increases dramatically, starting from a certain temperature (in the experiment, from 93°C).

Prior to performing the numerical study, we measured the specific heat and density of the hydrocarbon fluid: $c_{\rm f} = 1864$ J/(kg $\cdot$ K) and $\rho_{\rm f} = 954$ kg/m^3. The thermal conductivity at the initial temperature was $\lambda_{\rm f} = 0.125$ J/(kg $\cdot$ K). The viscosity of the hydrocarbon fluid was found as a function of temperature. The experiments showed that the object under study was nonfluid at temperatures $T < 35.5°$C and it was difficult to measure the rheological properties of the substance. That is why the study was performed in the temperature interval of 35.5–75°C. The resultant dependence of viscosity on temperature was approximated with the use of two exponents as

$$\eta(T) = \begin{cases} \eta_{01} \exp\left(-\gamma_1 (T - 35.5)\right), & 35.5\ °\mathrm{C} < T < 54.2\ °\mathrm{C}, \\ \eta_{02} \exp\left(-\gamma_2 (T - 54.2)\right), & 54.2\ °\mathrm{C} < T < 75°\mathrm{C}, \end{cases}$$

where $\eta_{01} = 1460$ Pa $\cdot$ sec is the viscosity of the hydrocarbon fluid at a temperature of 35.5°C, $\eta_{02} = 0.228$ Pa $\cdot$ sec is the viscosity of the hydrocarbon fluid at 54.2°C, $\gamma_1 = 0.497$ K^{-1} is the temperature coefficient in the range $T = 35.5$–$54.2°$C, and $\gamma_2 = 0.031$ K^{-1} is the temperature coefficient in the range $T > 54.2°$C.

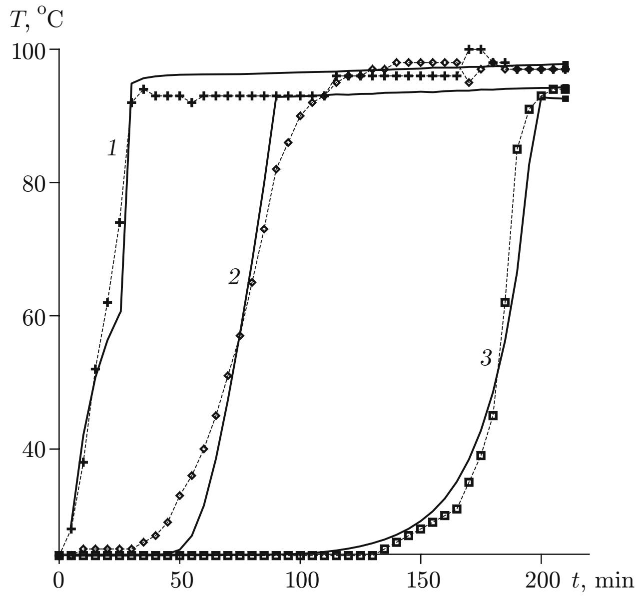

Fig. 2

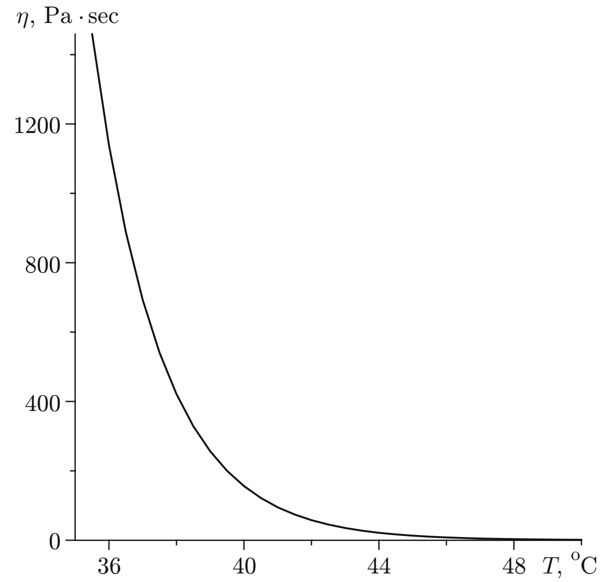

Fig. 3

Fig. 2. Variation of temperature in the fluid: the points and solid curves show the experimental and predicted data, respectively; curves 1, 2, and 3 refer to the positions of the first, second, and third thermocouples, respectively.

Fig. 3. Dynamic viscosity of the examined hydrocarbon fluid versus temperature.

Mathematical Model. Heating of the hydrocarbon fluid was modeled in axisymmetric geometry using the parameters of the experimental setup. We used a cylindrical coordinate system r, φ, z with the z axis directed upward along the reservoir centerline. From the viewpoint of mathematical modeling, the physical model is a multilayer medium. There are four layer along the coordinate r: the first one is $0 \leqslant r \leqslant R_1$ (to simplify the calculations, we assumed that the inductor filled by transformer iron was a continuous steel cylinder of radius R_1 and length l), the second layer is the air gap between the inductor and the inductor tube ($R_1 \leqslant r \leqslant R_2$), the third layer is the metal wall of the inductor tube ($R_2 \leqslant r \leqslant R_3$), and the fourth layer is the hydrocarbon fluid ($R_3 \leqslant r \leqslant R_4$). Here R_2 and R_3 are the inner and outer radii of the inductor tube, and R_4 is the radius of the reservoir containing oil sludge.

The laboratory model has also four layers along the coordinate z: the first one is the concrete bed at the bottom of the reservoir ($0 \leqslant z \leqslant Z_3$), the second layer is the inductor ($Z_3 \leqslant z \leqslant Z_3 + l$), the third layer is the air gap above the inductor up to the hydrocarbon-fluid surface in the reservoir ($Z_3 + l \leqslant z \leqslant Z_3 + Z_1$), and the fourth layer is the portion of the inductor tube sticking up above the hydrocarbon fluid ($Z_3 + Z_1 \leqslant z \leqslant Z_3 + Z_1 + Z_4$). Here Z_3 is the height of the concrete bed, l is the height of the inductor, Z_1 is the level to which the reservoir is filled by oil sludge, and Z_4 is the height of the inductor tube above the oil-sludge surface.

Formulation of the Problem. The inductor is constructed so that the energy of eddy currents generated by the inductance coil is released in the form of heat in the inductor-tube wall. This heat release occurs in a thin skin layer of the tube, but, because of a small thickness of the tube wall and a high thermal conductivity of the metal, the heat generated by the inductor instantaneously propagates and can be treated as a system of internal distributed sources of heat in the inductor-tube wall. The equation for heat conduction in the inductor-tube wall is

$$c_{\rm m}\rho_{\rm m}\,\frac{\partial T}{\partial t} = \frac{\lambda_{\rm m}}{r}\,\frac{\partial}{\partial r}\Big(r\,\frac{\partial T}{\partial r}\Big) + \lambda_{\rm m}\,\frac{\partial^2 T}{\partial z^2} + q \qquad \Big(q = \frac{N_0}{\pi(R_3^2 - R_2^2)l}\Big), \tag{1}$$

where $c_{\rm m}$, $\rho_{\rm m}$, and $\lambda_{\rm m}$ are the specific heat, density, and thermal conductivity of the metal, respectively, q is the density of the distributed sources of heat, and N_0 is the inductor power.

In all other regions under consideration (the air gap between the inductor and the inductor tube and the hydrocarbon fluid), the thermal conductivity of the medium is a variable quantity. In the first case, variation in heat conductivity is caused by possible air convection, and an additional factor in the second is the dependence of molecular thermal conductivity on temperature. Here the heat-conduction equations are

$$c_{\rm a}\rho_{\rm a}\,\frac{\partial T}{\partial t}=\frac{1}{r}\,\frac{\partial}{\partial r}\Big(\varepsilon_{\rm a}\lambda_{\rm a}r\,\frac{\partial T}{\partial r}\Big)+\frac{\partial}{\partial z}\Big(\varepsilon_{\rm a}\lambda_{\rm a}\,\frac{\partial T}{\partial z}\Big); \tag{2}$$

$$c_{\rm f}\rho_{\rm f}\,\frac{\partial T}{\partial t}=\frac{1}{r}\,\frac{\partial}{\partial r}\Big(\varepsilon_{\rm f}\lambda_{\rm f}(T)r\,\frac{\partial T}{\partial r}\Big)+\frac{\partial}{\partial z}\Big(\varepsilon_{\rm f}\lambda_{\rm f}(T)\,\frac{\partial T}{\partial z}\Big), \tag{3}$$

where $c_{\rm a}$, $\rho_{\rm a}$, and $\lambda_{\rm a}$ are the specific heat, density, and thermal conductivity of air, $c_{\rm f}$, $\rho_{\rm f}$, and $\lambda_{\rm f}$ are the specific heat, density, and thermal conductivity of the hydrocarbon fluid, and $\varepsilon_{\rm a}$ and $\varepsilon_{\rm f}$ are the coefficients of air and hydrocarbon-fluid convection, respectively.

In the calculations, we used a linear dependence of the thermal conductivity of the fluid on temperature (such a dependence is known to be acceptable for many materials [1]):

$$\lambda_{\rm f}(T)=\lambda_{\rm f,0}[1+b(T-T_0)].$$

Here $\lambda_{\rm f,0}$ is the thermal conductivity at $T=T_0$ and b is the temperature coefficient of thermal conductivity.

In the concrete bed, the heat-conduction equation is

$$c_{\rm con}\rho_{\rm con}\,\frac{\partial T}{\partial t}=\frac{\lambda_{\rm con}}{r}\,\frac{\partial}{\partial r}\Big(r\,\frac{\partial T}{\partial r}\Big)+\lambda_{\rm con}\,\frac{\partial^2 T}{\partial z^2}. \tag{4}$$

Here $c_{\rm con}$, $\rho_{\rm con}$, and $\lambda_{\rm con}$ are the specific heat, density, and thermal conductivity of concrete, respectively.

Boundary Conditions. At the outer side and upper surfaces of the inductor tube sticking up above the hydrocarbon fluid, the heat exchange with the ambient medium was assumed to obey the law of free convection in an unbounded space [1, 5]. Hence, the boundary conditions are

$$\alpha_{\rm v}(T(R_3,z,t)-T_0)=-\lambda_{\rm m}\,\frac{\partial T(R_3,z,t)}{\partial r}; \tag{5}$$

$$\alpha_{\rm h}(T(r,Z_3+Z_1+Z_4,t)-T_0)=-\lambda_{\rm m}\,\frac{\partial T(r,Z_3+Z_1+Z_4,t)}{\partial z}. \tag{6}$$

Here $\alpha_{\rm v}$ are the coefficient of heat transfer along the vertical wall, $\alpha_{\rm h}$ is the coefficient of heat transfer along the horizontal wall, and T_0 is the initial temperature of the medium equal to the temperature of ambient air around the experimental setup.

The oil-reservoir model is installed on a wooden pallet and its sides are insulated by foam plastic; therefore, the heat flux at the bottom of the laboratory model and at its sides is assumed to be zero:

$$T(r,z,0)=T_0; \tag{7}$$

$$\frac{\partial T(0,z,t)}{\partial r}=0,\qquad \frac{\partial T(R_4,z,t)}{\partial r}=0; \tag{8}$$

$$\frac{\partial T(r,0,t)}{\partial z}=0. \tag{9}$$

At the oil-sludge surface, the heat exchange with the ambient air also obeys the law of free convection:

$$\alpha_{\rm h}(T(r,Z_3+Z_1,t)-T_0)=-\lambda_{\rm f}\,\frac{\partial T(r,Z_3+Z_1,t)}{\partial z}. \tag{10}$$

At the remaining boundaries of the layers, continuity of temperature and heat fluxes, i.e., a boundary condition of the fourth kind, is adopted [1, 5].

The Nusselt number (Nu) depends on the Grashof (Gr) and Prandtl (Pr) numbers [1] as

$$\mathrm{Nu}=\alpha L/\lambda_{\rm a}=f(\mathrm{Gr_a},\mathrm{Pr_a});\qquad \mathrm{Gr_a}=\beta_{\rm a}gL^3\Delta T_{\rm a}/\nu_{\rm a}^2;\qquad \mathrm{Pr_a}=\nu_{\rm a}/a_{\rm a},$$

where α is the heat-transfer coefficient, L is the characteristic size, $\beta_{\rm a}=1/T$ is the coefficient of volume expansion of air, T is the absolute temperature, g is the acceleration of gravity, $\Delta T_{\rm a}$ is the temperature difference between the heated wall and the ambient air, and $\nu_{\rm a}$ and $a_{\rm a}$ are the kinematic viscosity and the thermal conductivity of air.

To determine the coefficient of heat transfer along the vertical wall, we use the equation

$$\mathrm{Nu} = 0.75(\mathrm{Gr_a Pr_a})^{0.25}(\mathrm{Pr_a}/\mathrm{Pr_w})^{0.25},$$

where the subscript "w" refers to the wall. For air and diatomic gases, the Prandtl number is almost independent of temperature; hence, $\mathrm{Pr_a}/\mathrm{Pr_w} = 1$. The characteristic size is the height of the portion of the tube sticking up above the hydrocarbon fluid: $L = Z_4$.

The characteristic sizes in the boundary conditions (6) and (10) are the tube and reservoir diameters, respectively.

The effective thermal conductivity $\lambda_{c,\mathrm{f}}$ of the hydrocarbon fluid is described by the formula [5]

$$\lambda_{c,\mathrm{f}} = \varepsilon_\mathrm{f}\lambda_\mathrm{f}.$$

In turn, the convection coefficient of the hydrocarbon fluid ε_f can be found from the following expressions:

$$\varepsilon_\mathrm{f} = \begin{cases} 0.105(\mathrm{Gr_f Pr_f})^{0.3}, & 10^3 < \mathrm{Gr_f Pr_f} < 10^6, \\ 0.40(\mathrm{Gr_f Pr_f})^{0.2}, & 10^6 < \mathrm{Gr_f Pr_f} < 10^{10}. \end{cases} \quad (11)$$

The Grashof and Prandtl numbers are

$$\mathrm{Gr_f} = \beta_\mathrm{f} g L^3 \Delta T_\mathrm{f}/\nu_\mathrm{f}^2, \qquad \mathrm{Pr_f} = \nu_\mathrm{f}/a_\mathrm{f},$$

where β_f is the coefficient of volume expansion of the hydrocarbon fluid, ΔT_f is the temperature difference between the heated wall of the inductor tube and the point with a coordinate r where the temperature $T_\mathrm{f} = 93°\mathrm{C}$ is reached, and ν_f and a_f are the kinematic viscosity and the thermal diffusivity of the hydrocarbon fluid.

Free convection in air also results in an enhanced thermal conductivity. The effective thermal conductivity of air $\lambda_{c,\mathrm{a}}$ is

$$\lambda_{c,\mathrm{a}} = \varepsilon_\mathrm{a}\lambda_\mathrm{a}.$$

The convection coefficient of air ε_a can be found from expressions similar to (11):

$$\varepsilon_\mathrm{a} = \begin{cases} 0.105(\mathrm{Gr_a Pr_a})^{0.3}, & 10^3 < \mathrm{Gr_a Pr_a} < 10^6, \\ 0.40(\mathrm{Gr_a Pr_a})^{0.2}, & 10^6 < \mathrm{Gr_a Pr_a} < 10^{10}. \end{cases}$$

The Grashof and Prandtl numbers are

$$\mathrm{Gr_a} = \beta_\mathrm{a} g L^3 \Delta T/\nu_\mathrm{a}^2, \qquad \mathrm{Pr_a} = \nu_\mathrm{a}/a_\mathrm{a},$$

where ΔT is the temperature difference between the inductor and the inductor tube. The characteristic size L here is the gap between the inductor and the inductor tube, i.e., $R_2 - R_1$.

Calculation Results. System (1)–(4) with the boundary conditions (5)–(10) was solved by the method of finite differences along variable directions by an implicit scheme. The parameters of the medium had the following values [1, 6]: $\lambda_\mathrm{m} = 45$ W/(m·K), $c_\mathrm{m} = 461$ J/(kg·K), $\rho_\mathrm{m} = 7900$ kg/m^3, $\lambda_\mathrm{a} = 0.02896$ W/(m·K), $c_\mathrm{a}\rho_\mathrm{a} = 1065$ J/(m^3·K), $\nu_\mathrm{a} = 1.897 \cdot 10^{-5}$ m^2/sec, $\lambda_\mathrm{con} = 0.279$ W/(m·K), $c_\mathrm{con}\rho_\mathrm{con} = 2.6 \cdot 10^6$ J/(m^3·K), and $\beta_\mathrm{f} = 3.9 \cdot 10^{-4}$ K^{-1}.

In the calculations, the unknown coefficients were varied to reach the best possible agreement between the predicted and experimental data, which allowed obtaining $b = 0.513$ K^{-1} and the maximum value of the convection coefficient equal to $\varepsilon_{c,\mathrm{f}} = 16$ for the hydrocarbon fluid (oil sludge) under study. The coefficient b, which characterizes the increase in molecular thermal conductivity with temperature, shows that the thermal conductivity increases by a factor of 40 as the temperature increases from 24 to 100°C. The experimental and predicted curves of temperature dynamics are compared in Fig. 2. The saturation of the curves above the temperature $T \approx 93°\mathrm{C}$ can be attributed to complete boiling of lighter fractions of oil and water contained in oil sludge. This explanation is confirmed by visual observations.

Figure 4 shows a three-dimensional representation of the temperature field in the reservoir at the time of 50 min. The reservoir bottom is parallel to the coordinate r. The center of the reservoir bottom lies at the point with the coordinate $r = 0$. The data in the figure can be explained as follows. There is an inductor installed at the center of the reservoir, which is not heated but transmits the energy to the working tube. A certain increase in temperature at the inductor coil occurs owing to the reverse heat transfer through air from the working tube. The

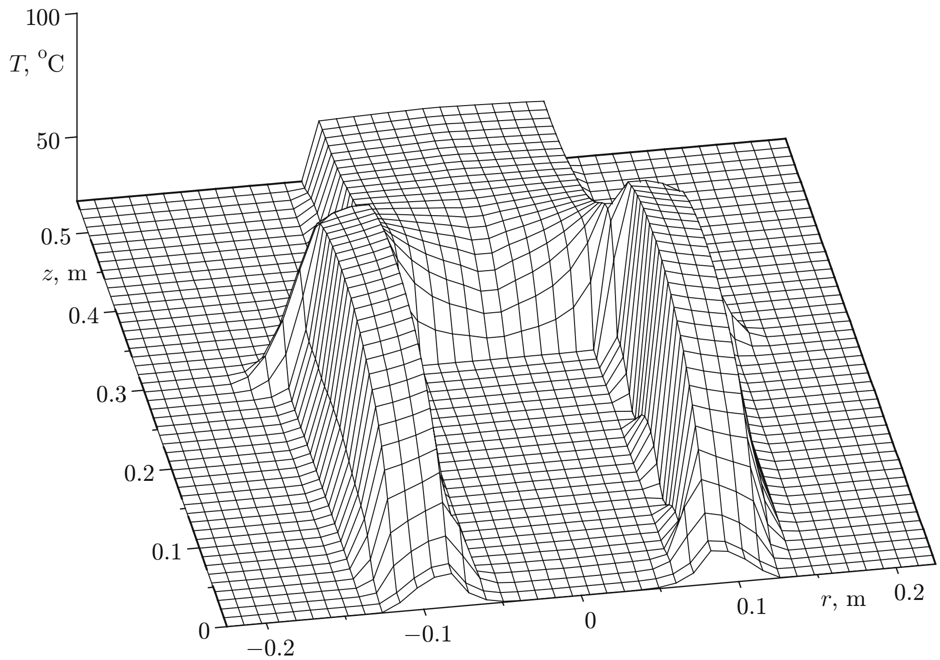

Fig. 4. Temperature field in the model reservoir with the hydrocarbon fluid at the time $t = 50$ min.

most heated part of the model is the tube at the level of the inductor center. Because of a strong temperature dependence of the thermal conductivity of the hydrocarbon fluid and free convection, two regions with disparate temperatures are formed in the medium: high-temperature region where the temperature is almost uniform because of high thermal conductivity of the substance and low-temperature region (with the temperature approximately equal to the initial value). These regions are separated by a narrow transitional region with intermediate values of temperature.

The anomalous behavior of thermal conductivity can be attributed to inhomogeneity of the heterogeneous hydrocarbon system under consideration. With the release of gases, their surfacing, and an increase in bubble size as the bubbles rise to the surface and the temperature increases, the hydrocarbon system resembles a porous medium. The effective thermal conductivity of porous bodies is known to be a conventional quantity [1]. In a hydrocarbon system, the "skeleton" is the liquid phase, and the pores are the released gas bubbles. As a result, the hydrocarbon fluid displays complex heat transfer with a combination of three heat-transfer mechanisms: thermal conduction, convection, and radiative heat transfer inside gas bubbles. After all three mechanisms come into operation at a temperature $T \approx 100°\mathrm{C}$ the effective heat-transfer coefficient of the hydrocarbon fluid becomes commensurable with the thermal conductivity of iron.

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